Department of Mechanics and Machine Design

Graphical Programming

Transfer function model

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Transfer function model

In the most general case, the output signal y(t) can be the differential equation of the higher order of the input signal u(t)

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) =$$

$$= b_m \frac{d^m u(t)}{dt^m} + b_{m-1} \frac{d^{m-1} u(t)}{dt^{m-1}} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t),$$

where a_i *for* $i = 0..n$,

b_i, *for* $i = 0..m$

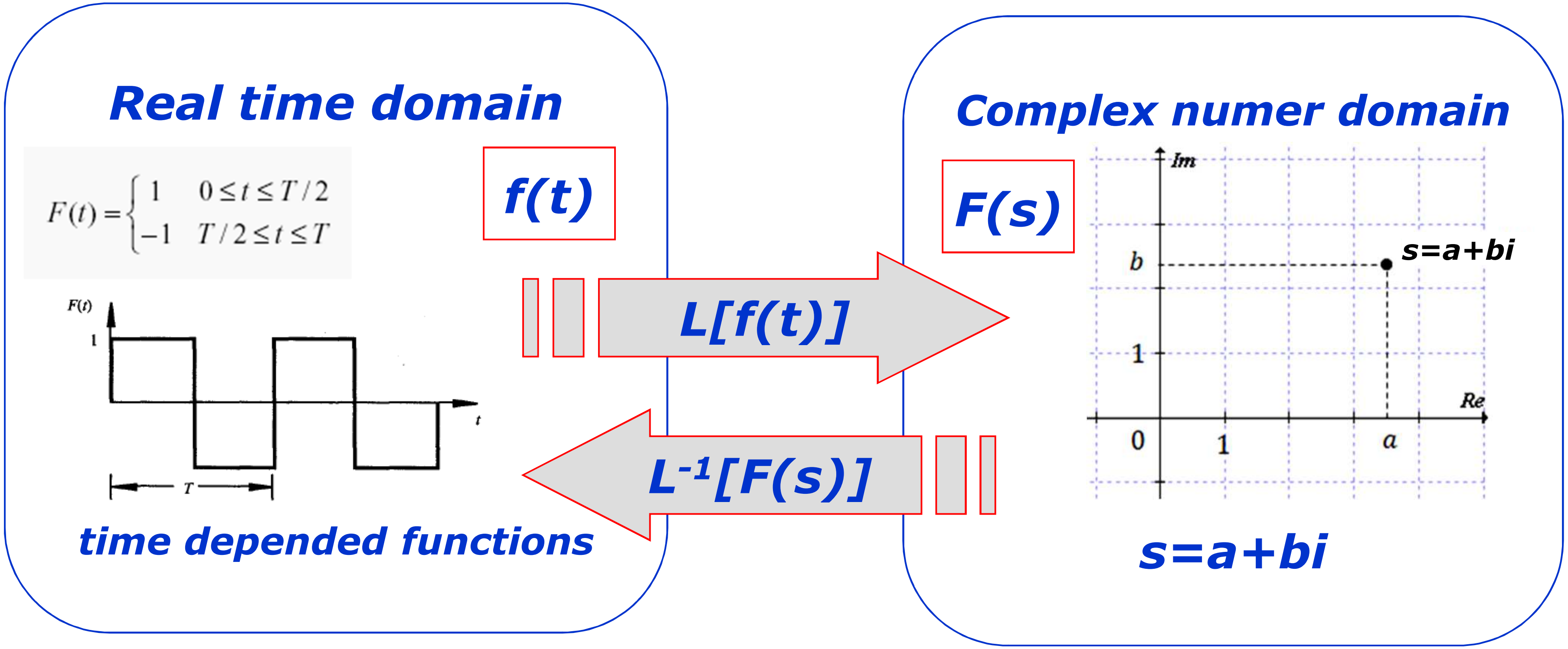
they are constant coefficients of the equation

Transfer function model

Transformation performing a certain function f(t) (the so-called ***original****) into the function of a complex variable f(s) (the so-called* ***image****),*

$$L[f(t)] = f(s) = \int_0^{\infty} f(t)e^{-st}dt$$

where: s∈C; C - set of complex numbers, s - complex number, t - time.



Linearity of the transform

$$L[A \cdot f(t) + B \cdot f(p)] = A \cdot L f(t) + B \cdot L f(p)$$

$$\text{e.g. } L[3 \cdot \sin(2t) + 7 \cdot e^{-3t}] = 3 \cdot L[\sin(2t) + 7L[e^{-3t}]$$

Simplified rules:

$$L[y(t)] = y(s)$$

$$L[y'(t)] = s \cdot y(s)$$

$$L[y''(t)] = s^2 \cdot y(s)$$

Laplace transfer of ODE:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) =$$

$$= b_m \frac{d^m u(t)}{dt^m} + b_{m-1} \frac{d^{m-1} u(t)}{dt^{m-1}} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t),$$

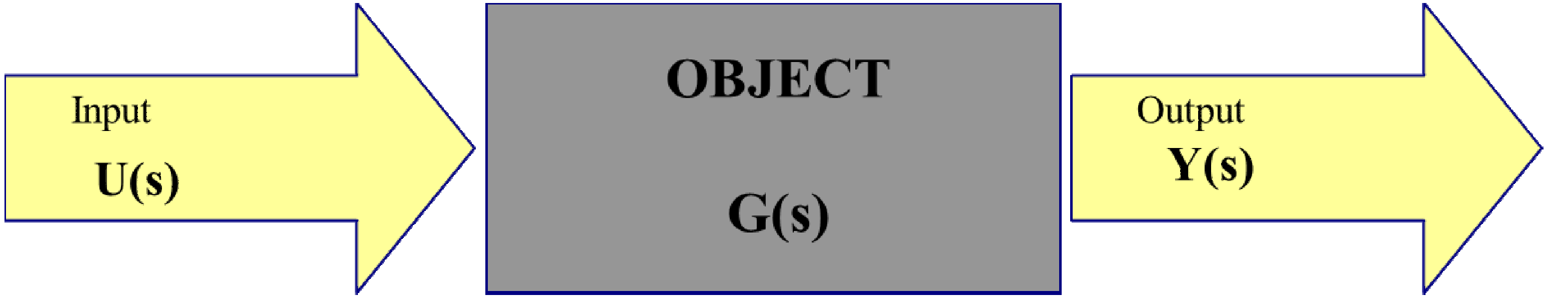
L[y(t), u(t)]

$$s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 = b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0$$

Transfer function model

*The function converting the input signal to the output (**object transmittance**) can be defined as:*

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$



Use known G(s) to calculate response Y(s) on input U(s):

$$Y(s) = G(s)U(s)$$

Transfer function model

Example:

$$\frac{dy}{dt} - 3y = 2x$$

$$L\left(\frac{dy}{dt} - 3y\right) = L(2x)$$

$$L\left(\frac{dy}{dt}\right) - 3L(y) = 2L(x)$$

$$L(y') - 3L(y) = 2L(x)$$

note:

$$L(y') = s \cdot y(s) \qquad 3L(y) = 3 \cdot y(s) \qquad 2L(x) = 2 \cdot x(s)$$

$$s \cdot y(s) - 3 \cdot y(s) = 2 \cdot x(s)$$

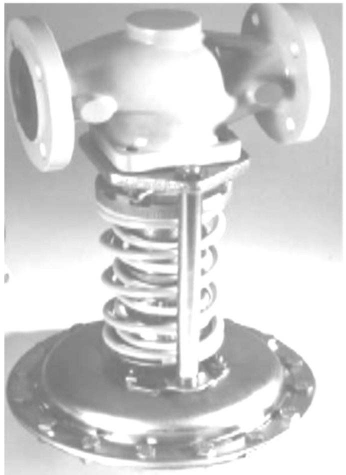
$$y(s) \cdot (s - 3) = 2 \cdot x(s)$$

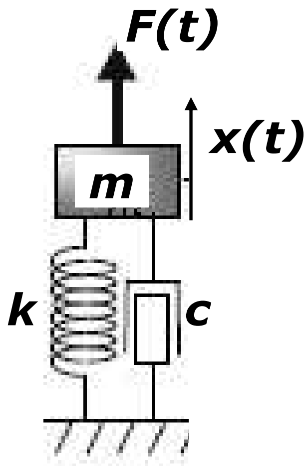
$$\frac{y(s)}{x(s)} = \frac{2}{s-3} = G(s)$$

Transfer function model

Example:

VALVE





F – spring force
m – moving mass
k – spring constant
c – damping
x – displacement

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

Transfer function model

Example:

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F(t)$$

$$L\left[m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx\right]=L[F(t)]$$

$$m\cdot L\left[\frac{d^2x}{dt^2}\right]+c\cdot L\left[\frac{dx}{dt}\right]+k\cdot L[x]=L[F(t)]$$

$$L\left[\frac{d^2x}{dt^2}\right]=s^2\cdot x(s) \qquad L\left[\frac{dx}{dt}\right]=s\cdot x(s) \qquad L[x]=x(s) \qquad L[F(t)]=F(s)$$

$$m\cdot s^2\cdot x(s)+c\cdot s\cdot x(s)+k\cdot x[s]=F(s)$$

$$x(s)\left(m\cdot s^2+c\cdot s+k\right)=F(s)$$

$$\frac{x(s)}{F(s)}=\frac{1}{m\cdot s^2+c\cdot s+k}=G(s)$$

Transfer function model

Transmittance is always in the form of a fraction:

$$G(s) = \frac{L(s)}{M(s)} = \frac{[\text{Numerator}]}{[\text{Denominator}]}$$

$$G(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_2 s^2 + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_2 s^2 + b_1 s + b_0}$$

$$\text{Numerator} = [a_0, a_1, a_2, \ldots a_{n-1}, a_n]$$

$$\text{Deno min ator} = [b_0, b_1, b_2, \ldots b_{n-1}, b_n]$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \qquad \Rightarrow \qquad a_0 = 1, b_0 = k, b_1 = c, b_2 = m$$

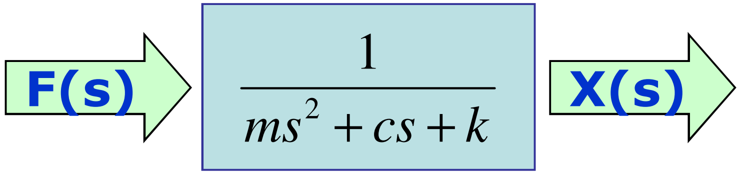
$$\text{Numerator} = [1]$$

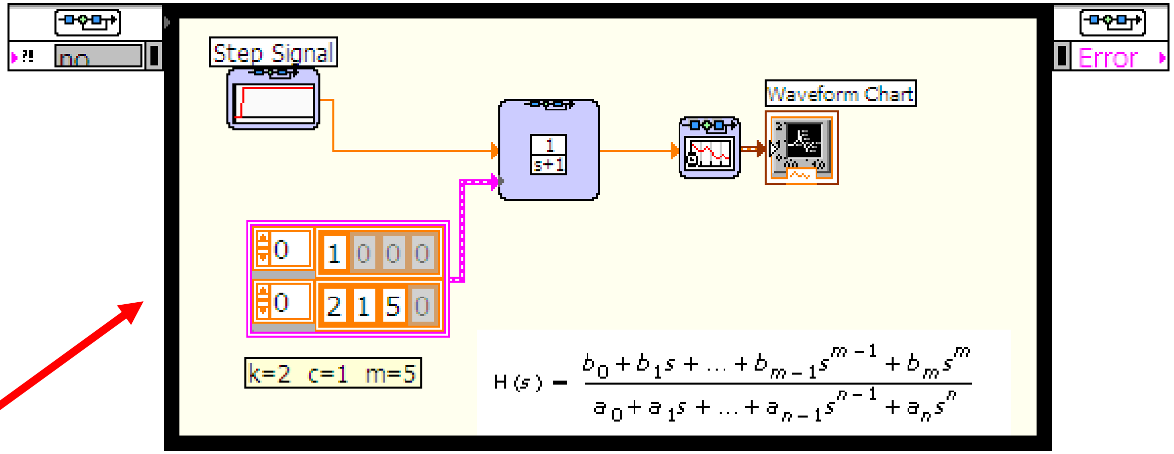
$$\text{Deno min ator} = [k, c, m]$$

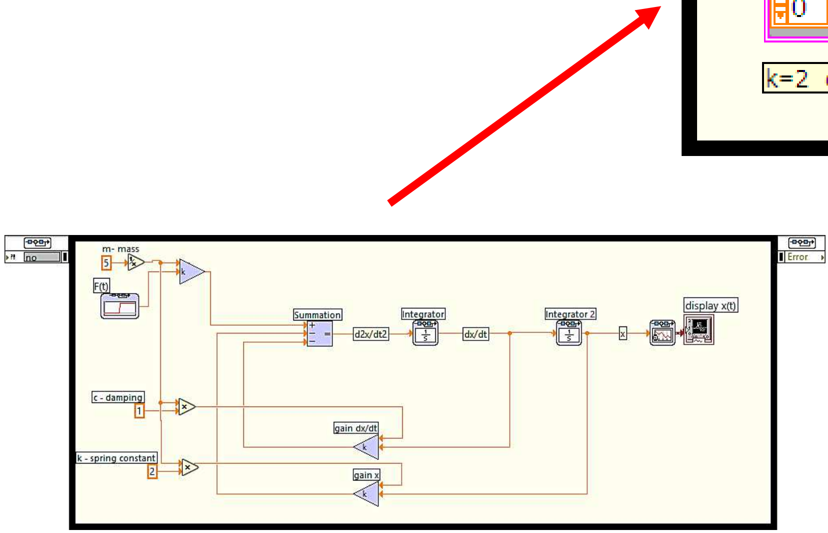
Transfer function model

Example:

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$







Transfer function model

Configuration window

In the Parameter source field in that dialog window you can select between Configuration page and Terminal. By selecting Configuration page you must define the numerator and denominator parameters of the transfer function on the dialog window.